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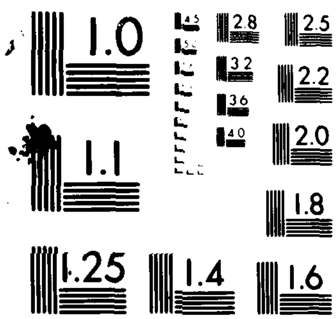

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Final Report

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U. S. Army Research Office

"STABILITY AND BIFURCATION OF UNSTEADY FLOWS"

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15 February 1982

Abstract

The work described herein concerns the determination of stability characteristics for time-dependent states of fluid flows or ordinary and partial differential equations in general. In particular nonlinear theories and simulations of bifurcation and stability are examined using energy methods, multiple-scale techniques and numerical procedures.

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Summary

The following topics have been considered during the time period stated.

1. Stability of quasi-periodic solutions. When a time-periodic solution of a system of ordinary or partial differential equations becomes unstable, the bifurcating solution may be periodic or quasi-periodic. In the latter case, the determination of the stability characteristics leads to the analysis of a quasi-periodic linear equation of Mathieu or Hill type. Such an analysis has been performed, Davis and Rosenblat (1980), for the case when the two frequencies are close together. The method of multiple scales is used to demark the stable from the unstable parametric regions.

2. Bifurcation of quasiperiodic solutions. In order to understand how quasi-periodic solutions bifurcate, some model equations were examined. The models involve sets of ordinary differential equations with a parameter λ, a pair of nonlinear coupled oscillators, whose linearized part has growth rates $\lambda \pm i\gamma$ and $\lambda \pm i\delta$. Hence, when λ passes through $\lambda_c = 0$, bifurcating solutions of frequencies γ and δ exist; this is essentially bifurcation from a multiple eigenvalue. When the systems are weakly nonlinear of amplitude ε, two cases arise. There is the "non-resonant" case in which $\gamma-\delta = O(1)$ as $\varepsilon \to 0$. This can be handled by a straightforward generalization of Hopf theory. There is the "resonant" case in which $\gamma-\delta = k\varepsilon$, $k = O(1)$, as $\varepsilon \to 0$ which gives rise to small divisor difficulties. Multiple scale techniques are used to overcome these and the gradual transformation from resonance to non-resonance is examined as $\rho \equiv 2(\delta-\gamma)/(\lambda-\lambda_c)$ varies. We find periodic solutions for $\rho < \rho_c$ and quasiperiodic solutions for $\rho > \rho_c$ where ρ_c is calculated. Hamiltonian and non-Hamiltonian systems show contrasting behavior. This work is completed and appears in Steen and Davis (1982a,b).

3. Energy stability of decelerating swirl flows. An infinitely long circular cylinder and its contained viscous fluid are rotating as a solid body. At time $t = 0$, the container begins decelerating. Three cases of shell motion are considered: impulsive, linear and exponential. Through viscous diffusion, an unsteady flow is set up. This flow can be centrifugally unstable near the boundary. An energy stability theory is set up to give lower bounds on the onset time t_0 and upper bounds on the decay time t_d. Here the onset time refers to when any instabilities present begin to grow, $t_0 > 0$. The decay time $t_d > 0$ refers to when any instabilities generated by the deceleration must begin to decay to zero. Since an energy theory is used, disturbances of arbitrary amplitude are allowed. The work is presented in Neitzel and Davis (1980a).

4. Nonlinear numerical simulation of spin-down instabilities. When a liquid-filled shell (a finite length circular cylinder) moves through the air, the shell motion is slowed by the action of a drag. The resulting fluid motion is called spin-down. Here, Ekman boundary layers at the ends acts as a pathway for the much augmented adjustment of the fluid motion to the new shell motion. The total flow field is axisymmetric and unsteady. This flow could also be centrifugally unstable leading to modified Taylor vortices which would in turn augment even further the rate of adjustment of the fluid motion to the cylinder motion. A finite-difference simulation of such instabilities has been completed. Here the Ekman boundary layers plus the side-wall layers must be resolved. The object here is to determine when such complicated three-dimensional unsteady flows become unstable, how the spin-down time is augmented by the instability, how the torque exerted by the field on the shell is modified, how the secondary

instability flow is characterized and how the enhanced mixing might influence the shell motion. This work is presented in Neitzel and Davis (1980b).

5. Stability and bifurcation in a modulated Burgers system. The stability of the null state for a nonlinear Burgers system is examined. The results include (i) an energy estimate for global stability for state involving arbitrary modulation in time, and (ii) an analysis of the bifurcation from the null state for slow modulations. For the slow modulations it is determined that the amplitude $A(t)$ of the bifurcated disturbance velocity satisfies a Landau-type equation with time-dependent growth rate $\theta(t)$. Particular attention is given to periodic and quasiperiodic modulations of the system, which lead to analogous behavior in $\theta(t)$. For each of these oscillatory-type modulations, it is found that $A^2(t)$ has the same long-time mean value as the unmodulated case, implying no alteration of the final mean kinetic energy. Applications to various fluid-dynamical phenomena are discussed. The work appears in Olmstead and Davis (1982).

6. Perturbed Hopf bifurcation. A study is being made of systems of ordinary differential equations exhibiting Hopf bifurcation in the presence of time-periodic coefficients. The model is chosen to simulate the characteristics of the bifurcation of time-modulated shear flows. Particular attention is paid to the behavior of the system near points of resonance, particularly sub-harmonic resonance.

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